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Mechanics

Pattern of Changes in Equivalent Rigidity of Nonlinear Mechanical Systems

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ABSTRACT. The theoretically predicted and experimentally determined regularity of change of the equivalent rigidity of nonlinear elements, when connected in series in static and dynamic systems, is studied. © *2009 Bull. Georg. Natl. Acad. Sci.*

Key words: *equivalent rigidity, nonlinear mechanical systems.*

According to the classical mechanics concepts, equivalent rigidity of the elastic and elastic-damping systems (both linear and nonlinear ones) when connected in series and in parallel within a system, is determined by recognized laws:

$$K = K_1 + K_2 + \ldots + K_i \quad \text{(when connected in parallel)}$$

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \ldots + \frac{1}{K_i} \quad \text{(when connected in series)}$$

$$(i = 3, 4, \ldots)$$

To this very day, researchers used to believe the above laws to be also true for nonlinear systems exposed to static or dynamic cycles of compression and loading. To illustrate this statement, let us take two springs with rigidity respectively K_1 and K_2 and connect them first in parallel and then in series. Let us determine the equivalent rigidity K of the system in each case.

If in the first case each of the springs is exposed to the same weight P, then elongation of each of the springs taken separately will make

$$x_1 = \frac{P}{K_1}; \qquad x_2 = \frac{P}{K_2}.$$

Total static displacement of the weight will make

$$x = x_1 + x_2 = \frac{P}{K_1} + \frac{P}{K_2}.$$

Equivalent rigidity of the system can be expressed by the formula

$$K = \frac{P}{x}, \quad or \quad K = \frac{K_1 K_2}{K_1 + K_2}, \quad \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}.$$

In the other case, if the value P_1 represents a stretching force applied to the upper spring with its rigidity of K_1

while P_2 is a compressive force applied to the lower spring with its rigidity of K_2, then both these forces are predetermined by the action of weight P. Naturally, the length of each of the springs will change by the same value, i.e.

$$Y_0 = \frac{P_1}{K_1} = \frac{P_2}{K_2} = \frac{P}{K}. \quad K = \frac{dF}{dx} = tg\alpha = const.$$

Equivalent rigidity of the system

$$K = K_1 + K_2.$$

Determination of the equivalent rigidity of a nonlinear system within which elastic or elastic-damping elements of nonlinear characteristics (both softening and rigid ones) are connected, represents a particular problem. As a result of interaction of the above elements, there appears a so-called "marginal zone" the characteristics of which are different from the theoretical characteristic of the system, that is why the afore-said law of determination of the equivalent rigidity of the system as a whole is violated.

The coefficient of rigidity of both linear and nonlinear mechanical systems is determined by formula $K=dF/dx$, so for the systems with linear characteristic, the rigidity is not changed and is equal to

$$K = \frac{dF}{dx} = tg\alpha,$$

while for systems with nonlinear characteristics, the rigidity will change dependent on the deformation value and is expressed as follows:

$$K_1 = \left.\frac{dF}{dx}\right|_{x=x_1} = tg\beta; \; K_2 = \left.\frac{dF}{dx}\right|_{x=x_1} = tg\gamma; \quad tg\beta < tg\gamma, \; (\beta < \gamma).$$

Respectively, in the first case, the equivalent rigidity of a system will theoretically be equal to:

$$K = \frac{tg^2\alpha}{tg2\alpha\left(1 - tg^2\alpha\right)},$$

while in the other case it will make

$$K = \frac{tg\beta \cdot tg\gamma}{tg\left(\beta+\gamma\right)\left(1 - tg\beta \cdot tg\gamma\right)}.$$

For quantitative evaluation of the violation of the law of determination of the equivalent rigidity of nonlinear elements connected in series, a special universal test workbench equipped with a pulsator 4DM-PU100000 was used. As stated as a result of the experiment, the equivalent coefficient of rigidity of nonlinear elements differs from that of the theoretic rigidity value and is determined by formula:

$$K = \frac{tg\beta \cdot tg\gamma}{\nu_1 \cdot tg\gamma + \nu_2 \cdot tg\beta}.$$

Correspondingly, $\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2}, \; \nu_1 < \nu_2.$

If the number of elements connected within a system is n, then the equivalent coefficient of rigidity is computed by formula:

$$\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2} + \ldots + \frac{\nu_n}{K_n},$$

where $K_1, K_2, \ldots, K_n$ represent theoretical coefficients of rigidity of corresponding elements; at that,

$$\nu_1 < \nu_2 < \ldots < \nu_n.$$

Numerical values of the coefficients $\nu_1 < \nu_2 <, \ldots, < \nu_n$ for different nonlinear elastic and elastic-damping elements are determined in experimental way.

For elements made of technical rubber, metallic rubber as well as for elastic-damping elements with hysteretic characteristic of elliptic type, the coefficient of rigidity is determined as follows:

$\beta = 55^0$, $K_1 = tg\beta$, $F = 100H$, $K_1 = 20H/mm$.

$\gamma = 40^0$, $K_2 = tg\gamma$, $F = 100H$, $K_2 = 12.5H/mm$. $K = 13H/mm$.

The experimental equivalent coefficient of rigidity has the following value:

$$K = 9.1H/mm, \text{ at that } \nu_1 = 0.65, \quad \nu_2 = 0,7.$$

For elements of a nonlinear spring and elements with nonlinear characteristic of hyperbolic type it is determined that

$$\beta = 55^0, \quad F = 120H, \quad K_1 = 25H/mm.$$

$$\gamma = 40^0, \quad F = 110H, \quad K_2 = 20H/mm.$$

Theoretical coefficient $K = 12.2H/mm$

The experimentally obtained coefficient $K = 15.2H/mm$, at that $\nu_1 = 0.75$, $\nu_2 = 0.8$

As stated on the basis of findings of multiple experimental studies, numerical values of the coefficients ν_i (i=1,2,…,n), dependent on the location of the elements and in case of their connection in the system in series will change within $0.5 \le \nu_i \le 0.8$ (i=1,2,…,n).

Formulae for determining deformations, tensions and equivalent rigidity of nonlinear mechanical elements, when connected in series and in parallel within a system are established experimentally.

Designations in the formulae:

E – dynamic elasticity module, H/m^2; G – module displacement, H/m^2; s - normal tensions, H/m^2;

t - touch tensions displacement, H/m^2; K – linear equivalent rigidity, H/m; S – area width dissection, m^2;

e– relative deformation displacement; q – relative deformation compressed; F – external load, H.

Load	Width conical springs (K_1,K_2) – sizes $2r_2 \times 2r_1 \times$h,
Statical and prolonged dynamical	compressed with power F on value Δ $\varepsilon = \frac{\Delta}{h}$; $\Delta = \frac{Fh}{ES}$; $K_1 = \frac{E_1 S_1}{h}$; $K_2 = \frac{E_2 S_2}{h}$; $\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2}$; $0.5 \le \nu_1 < \nu_2 \le 0.8$; $\sigma_1 = \frac{F}{S_1}$; $\sigma_2 = \frac{F}{S_2}$. Displacement with power F on value Δ $q = \frac{\Delta}{h}$; $\Delta = \frac{Fh}{G \cdot S}$; $K_1 = \frac{G_1 S_1}{h}$; $K_2 = \frac{G_2 S_2}{h}$. $\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2}$; $0.6 \le \nu_1 < \nu_2 \le 0.8$; $\tau_1 = \frac{\sqrt{3}F}{S_1}$; $\tau_2 = \frac{\sqrt{3}F}{S_2}$

Load	Nonlinear elements with absorption energy – sizes ($2r_2$-$2r_1$)l,
Statical and prolonged dynamical	compressed with power F on value Δ $K_1 = \frac{2\pi l E_1}{\ln \frac{r_2}{r_1}}$; $K_2 = \frac{2\pi l E_2}{\ln \frac{r_2}{r_1}}$; $\varepsilon = \frac{\Delta}{l}$; $\Delta = \frac{F}{2\pi E l} \ln \frac{r_2}{r_1}$; $E = E_1 + E_2$; $\sigma_1 = \frac{F}{\pi(r_2 - ltg\alpha)^2}$; $\sigma_2 = \frac{F}{\pi(r_1 - ltg\alpha)^2}$; $\alpha = arctg \frac{r_2}{r_1}$; $\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2}$; $0.55 \le \nu_1 < \nu_2 < 0.7$. Displacement with power F on value Δ $K_1 = \frac{2\pi l G_1}{\ln \frac{r_2}{r_1}}$; $K_1 = \frac{2\pi l G_2}{\ln \frac{r_2}{r_1}}$; $q = \frac{\Delta}{l}$; $\Delta = \frac{F}{2\pi G l} \cdot \ln \frac{r_2}{r_1}$; $G = G_1 + G_2$; $\tau_1 = \frac{F}{2\pi r_1 l}$; $\tau_2 = \frac{F}{2\pi r_2 l}$; $\frac{1}{K} = \frac{\nu_1}{K_1} + \frac{\nu_2}{K_2}$; $0.65 \le \nu_1 < \nu_2 \le 0.8$;

The theoretically predicted and experimentally determined regularity of change of the equivalent rigidity of nonlinear elements, when connected in series in static and dynamic systems, has cardinally changed the present-day concept regarding properties of elastic and elastic-damping elements of nonlinear characteristic. Hence, an earlier unknown regularity of change of the equivalent rigidity of nonlinear mechanical systems has been discovered and recognized as an item of discovery (positive authorization by the Russian Agency for Patents and Trade Marks).

Scientific and practical value of the research data lies in their application in solving a great many actual engineering problems, in particular for developing new vibro-damping devices for mining machinery operated in difficult geological conditions as well as for creation of durable and steady dynamic systems with certain life resource.

The above will result in a considerable increase in the capacity and reliability of machinery and ecologically, in a significant rise of working conditions of the operation personnel.

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