Mechanics

Study of the Influence of Clearances on the Dynamical Load of Machine Systems

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ABSTRACT. A generalized method of calculation of dynamic characteristics in transmissions of machine aggregates at different stages of machine starting on power transmissions of the main electrical drive with account of the co-existence of clearances is prepared. Analytical dependence between the sizes of gaps and additional forces (of shock character) caused by their influence is determined. © *2010 Bull. Georg. Natl. Acad. Sci.*

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The formation of clearances in main driving lines of mechanical systems is due to presence of spindles, gear and helical joints in transmissions of machine aggregates. Magnitudes of clearances are in direct proportional dependence to the life-time of these machines: clearances cause generation of percussive forces; which in their turn are sources of dynamical loads and they lead to accelerated fatigue of main units of machine aggregates (spindle, gear, cages, reducing gears and other details). Additional dynamical loads caused by clearances may exceed several times the static forces generated by the basic technologic regimes. Influence of clearances is especially significant for aggregates operating in reverse regimes, namely for rolling mills and especially for sheet rolling machines of high efficiency, where seizure of metal by rollers is realized at high speeds and always is characterized by significant percussive phenomena.

It is evident that new rolling machines must be designed considering additional impact forces caused by clearances. Unfortunately, reference material is not available in technical literature till today, which would enable us to establish analytical relationship between magnitudes of clearances and additional impact forces caused by them.

The present research aims to eradicate this shortcoming to a certain extent. Consider dynamical loads in transmissions with clearances at starting of a machine, for the case when reduced calculating scheme of a machine aggregate is a three-mass mechanical system (Fig. 1).

The following designations are adopted in the drawing: I_1 – moment of inertia of the electric motor rotor; I_2 – moment of inertia of intermediate masses (gear cage, reducing gear, flywheels, etc.) reduced to the electro motor shaft; I_3 – mass moment of inertia of the machine aggregate rollers, reduced to the electro motor shaft; Δ_1, Δ_2 – reduced radial clearances in corresponding sections.

We assume that mass I_1 is acted on by driving moment $M_1 = const$ and other masses – by corresponding moments of resistance M_2 and M_3.

At the first stage, when the electromotor rotor starts motion, other masses
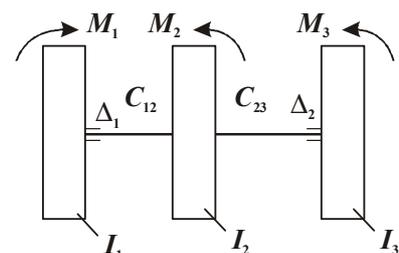

Fig. 1.

of the system under consideration are immobile. In this case the differential equation of movement has the simplest form:

$$I_1 \frac{d^2 \varphi}{dt^2} = M_1 .$$

(1)

At constant driving moment of the electromotor and zero initial conditions, expressions for the rotor angular velocity ω_1 and angular displacement φ_1 are found from equation (1):

$$\omega_1 = \varepsilon_0 t , \quad \varphi_1 = \varepsilon_0 t^2 / 2 ,$$

(2)

where $\varepsilon_0 = \dfrac{M_1}{I_1}$ is the rotor angular acceleration at constant acceleration of the system. At passing the clearance of

magnitude Δ_1 in the time interval T_{01}, the electromotor is accelerated up to certain magnitude ω_{1init}.

If we assume that $t = T_0$ and $\varphi_1 = \Delta_1$ for equation (2), then we obtain an expression defining ω_{1init}:

$$\omega_{1init} = \sqrt{\frac{2M_1 \Delta_1}{I_1}} = \sqrt{2\varepsilon_0 \Delta_1} .$$

(3)

The process of passing the clearance is determined by elastic percussion at the end of the first stage, since mass I_2 was immobile.

At the second stage, which begins after closing of the clearance in the first elastic section, differential equations of movement of the system will have the form (At this stage, mass I_2 and elastic section connected to it, begin movement):

$$\left. \begin{array}{l} I_1 \dfrac{d^2 \varphi_1}{dt^2} + C_{12}(\varphi_1 - \varphi_2) = M_1 \\[2mm] I_2 \dfrac{d^2 \varphi_2}{dt^2} - C_{12}(\varphi_1 - \varphi_2) = M_2 \end{array} \right\}$$

(4)

with the following initial conditions:

$$t_1 = 0; \quad \varphi_1 = \varphi_2 = 0 ; \quad \frac{d\varphi_1}{dt_1} = \omega_{1init}; \quad \frac{d\varphi_2}{dt_1} = 0$$

(5)

After solution of differential equations (4), initial conditions (5) we determine the magnitude of $\varphi_2 , \omega_1 , \omega_2 ,$ $M_{12} = C_{12}(\varphi_1 - \varphi_2)$.

$$\left. \begin{array}{l} \varphi_1 = \dfrac{M_a}{I_1 p^2}(1 - \cos p t_1) + \dfrac{I_2 \omega_{1init}}{(I_1 + I_2)p} \sin p t_1 + \dfrac{I_2 \omega_{1init}}{I_1 + I_2} t_1 + \varepsilon_{ar}^{(2)} \\[4mm] \varphi_2 = -\dfrac{M_a}{I_2 p^2}(1 - \cos p t_1) - \dfrac{I_1 \omega_{1init}}{(I_1 + I_2)} \sin p t_1 + \dfrac{I_1 \omega_{1init}}{I_1 + I_2} t_1 + \varepsilon_{ar}^{(2)} \dfrac{t_1^2}{2} \\[4mm] \omega_1 = \dfrac{M_a}{I_1 p} \sin p t_1 + \dfrac{I_2 \omega_{1init}}{I_1 + I_2} \cos p t_1 + \dfrac{I_1 \omega_{1init}}{I_1 + I_2} + \varepsilon_{ar}^{(2)} t_1 \\[4mm] \omega_2 = \dfrac{M_a}{\theta_2 p} \sin p t_1 - \dfrac{I_1 \omega_{1init}}{I_1 + I_2} \cos p t_1 + \dfrac{I_1 \omega_{1init}}{I_1 + I_2} + \varepsilon_{ar}^{(2)} t_1 \\[4mm] M_{12} = M_a (1 - \cos p t_1) + \dfrac{\omega_{1init} C_{12}}{p} \sin p t_1 \end{array} \right\} ,$$

(6)

where $M_a = \dfrac{I_2 M_1 + I_1 M_2}{I_1 + I_2}$; $\varepsilon_{av}^{(2)} = \dfrac{M_1 - M_2}{I_1 + I_2}$ is average acceleration after the closing of the first clearance; p is

partial frequency of proper oscillations, $p = \sqrt{\dfrac{C_{12}(I_1 + I_2)}{I_1 I_2}}$.

The above given expressions (6) enable us to determine the basic characteristics of the drive dynamical system after closing of the first clearance, when masses of the mechanical system perform complex rotary and vibratory movements. This latter is caused by the elasticity of the unit connecting masses I_1 and I_2.

The last expression in formulae (6) determines moment f elastic forces in the period of closing of the first clearance. The first term of this expression determines the moment of elastic forces caused by external forces acting on the system, and the second term – additional dynamical loads (of percussive character) generated by presence of the clearance. The second component of the moment of elastic forces can be analytically determined as follows:

$$M_{12} = \sqrt{2 M_1 I_1 C_{12}\left(\dfrac{I_2}{I_1 + I_2}\right)} \sin p t_1 \ . \tag{7}$$

Determine the dynamic coefficient of the two-mass mechanical system considering the influence of the clearance. Its value is calculated by the ratio of maximum moment of elastic forces to moment M_a (moment M_a is a static load in the elastic unit in the case of lack of clearance):

$$K_d = 1 + \sqrt{1 + (\dfrac{\omega_{1init} c_{12}}{M_a p})^2} = 1 + \sqrt{1 + \dfrac{2 M_1 n_2 c_{12}}{M^2_a}\Delta_1} \ , \tag{8}$$

where $n_2 = \dfrac{I_2}{I_1 + I_2}$.

Reduction of dynamical loads caused by percussions can be practically reached by qualitative fabrication and mounting of details of connecting units. Minimization of percussive loads caused by the influence of clearances can be realized also with the help of regulation of the electromotor starting and braking according to a certain optimal law.

Formula (8) determines maximum value of the dynamic coefficient of two-mass mechanical systems. After closing of the first clearance the second mass of the mechanical system performs complex movement (rotary and vibratory). After a certain interval of time T_{02} , the second mass will pass clearance Δ_2 and close the driving system on the third mass. Time T_{02} of passing of the clearance is calculated by solution of the transcendental equation and substitution of magnitudes $\varphi_2 = \Delta_2$ and $t_1 = T_{02}$ in expression (6), taking into account the uniformly accelerated and vibratory movement of mass I_2:

$$\Delta_2 = \dfrac{M_a}{I_2 p^2}(1 - \cos p T_{02}) - \dfrac{I_1 \omega_{1init}}{(I_1 + I_2)p}\sin p T_{02} + \dfrac{I_1 \omega_{1init}}{I_1 + I_2}T_{02} + \varepsilon_{ar}^{(2)}\dfrac{T_{02}^2}{2} \ . \tag{9}$$

Namely, parameters defining inertness and stiffness of rolling mills are chosen in the way that in such system $p \geq 100\dfrac{1}{\sec}$. This enables us to neglect the influence of the first and second terms in equation (9). Then T_{02} is calculated by solving the following quadratic equation:

$$\varepsilon_{ar}^{(2)}T_{02}^2 + \dfrac{2 I_1 \omega_{1init}}{I_1 + I_2}T_{02} - 2\Delta_2 = 0, \tag{10}$$

which has one root:

$$T_{02} = \left\{ -\frac{I_1 \omega_{1init}}{I_1 + I_2} + \sqrt{\frac{I_1^2 \omega^2_{1init}}{(I_1 + I_2)^2} + 2\Delta_2 \varepsilon_{ar}^{(2)}} \right\} \frac{1}{\varepsilon_{ar}^{(2)}} . \tag{11}$$

Approximate formulae (10) and (11) for calculation of T_{02} can be used for solution of the exact transcendental equation (9), from which we find the true magnitude of T_{02}.

At the third stage, after closing the clearance Δ_2, the differential equations of the system movement will have the form:

$$\left. \begin{aligned} I_1 \frac{d^2\varphi_1}{dt_2^{\,2}} + C_{12}(\varphi_1 - \varphi_2) &= M_1 \\ I_2 \frac{d^2\varphi_2}{dt^2} - C_{12}(\varphi_1 - \varphi_2) + C_{23}(\varphi_2 - \varphi_3) &= -M_2 \\ I_3 \frac{d^2\varphi_3}{dt^2} - C_{23}(\varphi_2 - \varphi_3) &= -M_3 \end{aligned} \right\} . \tag{12}$$

We present equations (12) in another form:

$$\left. \begin{aligned} I_1 \ddot{\varphi}_1 + I_2 \ddot{\varphi}_2 + I_3 \ddot{\varphi}_3 &= M_1 - M_2 - M_3; \\ \ddot{\varphi}_1 - \ddot{\varphi}_2 + \frac{C_{12}(I_1 + I_2)}{I_1 I_2}(\varphi_1 - \varphi_2) - \frac{C_{23}}{I_2}(\varphi_2 - \varphi_3) &= \frac{M_1}{I_1} + \frac{M_2}{I_2}; \\ \ddot{\varphi}_2 - \ddot{\varphi}_3 + \frac{C_{23}(I_2 + I_3)}{I_2 I_3}(\varphi_2 - \varphi_3) - \frac{C_{12}}{I_2}(\varphi_1 - \varphi_2) &= -\frac{M_2}{I_2} + \frac{M_3}{I_3} \end{aligned} \right\} . \tag{13}$$

We use the rule of transformation of coordinates:

$$\left. \begin{aligned} \varphi_1 &= \bar{\xi} + \frac{I_2 + I_3}{I_0}\theta_{12} + \frac{I_3}{I_0}\theta_{23} \\ \varphi_2 &= \bar{\xi} - \frac{I_1}{I_0}\theta_{12} + \frac{I_3}{I_0}\theta_{23} \\ \varphi_3 &= \bar{\xi} - \frac{I_1}{I_0} - \frac{I_1 + I_2}{I_0}\theta_{23} \\ (\dot{\xi} &= U) \end{aligned} \right\} , \tag{14}$$

where $I_0 = I_1 + I_2 + I_3$.

After such transformation equation (14) assumes the form

$$\left.\begin{aligned}
\dot{U} &= \varepsilon_{ar}^{(3)} \\
\ddot{\theta}_{12} &+ \frac{C_{12}(I_1 + I_2)}{I_1 I_2}\theta_{12} - \frac{C_{23}}{I_2}\theta_{23} = M_a\frac{I_1 + I_2}{I_1 I_2} \\
\ddot{\theta}_{23} &+ \frac{C_{23}(I_2 + I_3)}{I_2 I_3}\theta_{23} - \frac{C_{12}}{I_2}\theta_{12} = M_b\frac{I_1 + I_2}{I_1 I_2}
\end{aligned}\right\}, \tag{15}$$

where $\varepsilon_{av}^{(3)} = \dfrac{M_1 - M_2 - M_3}{I_0}$ is average acceleration of the system after closing of clearance Δ_2;

$$M_b = \frac{M_3 I_2 - M_2 I_3}{I_2 I_3}.$$

Initial conditions of differential equations (15) will be determined considering the fact that at initial moment of closing of the second clearance connecting section (unit) of the first and second masses is deformed:

$$\theta_{12}(0) = \theta_{12}(T_{02}) = \frac{M_a}{C_{12}}(1 - \cos pT_{02}) + \frac{\omega_{1init}}{p}\sin pT_{02} \tag{16}$$

and the connecting unit of the second and third masses is not deformed:

$$\theta_{23}(0) = \theta_{23}(T_{02}) = 0. \tag{17}$$

Angular velocities and displacements of masses I_1 and I_2 at initial moment of closing of the second clearance are determined by the following formulae:

$$\left.\begin{aligned}
\omega_1(T_{02}) &= \frac{M_a}{I_1 p}\sin pT_{02} + \frac{I_2}{I_1 + I_2}\omega_{1init}\cos pT_{02} + \frac{I_1}{I_1 + I_2}\omega_{1init} + \varepsilon_{av}^{(2)}T_{02} \\
\omega_2(T_{02}) &= -\frac{M_a}{I_2 p}\sin pT_{02} - \frac{I_1}{I_1 + I_2}\omega_{1init}\cos pT_{02} + \frac{I_1}{I_1 + I_2}\omega_{1init} + \varepsilon_{av}^{(2)}T_{02} \\
\varphi_1(T_{02}) &= -\frac{M_a}{I_1 p^2}(1 - \cos pT_{02}) + \frac{I_2}{(I_1 + I_2)p}\omega_{1init}\sin pT_{02} + \frac{I_1}{I_1 + I_2}\omega_{1init}T_{02} + \varepsilon_{av}^{(2)}\frac{T_{02}}{2} \\
\varphi_2(T_{02}) &= -\frac{M_a}{I_2 p^2}(1 - \cos pT_{02}) - \frac{I_2}{(I_1 + I_2)p}\omega_{1init}\sin pT_{02} + \frac{I_1}{I_1 + I_2}\omega_{1init}T_{02} + \varepsilon_{av}^{(2)}\frac{T_{02}}{2} = \Delta_2
\end{aligned}\right\}. \tag{18}$$

Then on the basis of formula (18) we have:

$$\dot{\theta}_{12}(0) = \dot{\theta}_{12}(T_{02}) = \omega_1(T_{02}) - \omega_2(T_{02}) = \frac{M_a(I_1 + I_2)}{pI_1 I_2}\sin pT_{02} + \omega_{1init}\cos pT_{02}, \tag{19}$$

$$\dot{\theta}_{23}(0) = \dot{\theta}_{23}(T_{02}) = \omega_2(T_{02}). \tag{20}$$

can be determined through φ_1, φ_2 and φ_3 from expressions (14)

$$\bar{\xi} = \frac{1}{I_0}\left(I_1\varphi_1 + I_2\varphi_2 + I_3\varphi_3\right). \tag{21}$$

Considering that

$$U = \dot{\xi}, \tag{22}$$

we can write

$$U(T_{02}) = \dot{\xi}(T_{02}) = \frac{1}{I_0}\left[I_1\omega_{1init} + \left(I_1 + I_2\right)\varepsilon_{ar}^{(2)}T_{02}\right]. \tag{23}$$

Thus, at the moment of closing of the second clearance $t_2 = 0$, initial conditions (17,18,20,21,23) are observed by the system of differential equations.

By integration of the first equation of the system (15) we have

$$U(t_2) = \varepsilon_{av}^{(3)}t_2 + U(T_{02}). \tag{24}$$

Omitting intermediate transformations and considering the initial conditions (17,18,20,21) we write the solution of the second and third differential equations of system (15):

$$
\begin{aligned}
M_{12}(t_2) = C_{12}\theta_{12}(t_2) = \frac{1}{p_2{}^2 - p_1{}^2}\Bigg\{ & \left[C_{12}\theta_{12}(T_{02})(p_2{}^2 - p_{12}{}^2) + p_{12}{}^2 M_a - p_2{}^2 \frac{I_2 + I_3}{I_0}m_{12}\right]\cos p_1 t_2 - \\
& -\left[C_{12}\theta_{12}(T_{02})(p_1{}^2 - p_{12}{}^2) + p_{12}{}^2 M_a - p_1{}^2 \frac{I_2 + I_3}{I_0}m_{12}\right]\cos p_2 t_2 + \frac{1}{p_1}\Big[C_{12}\dot{\theta}_{12}(T_{02})(p_2{}^2 - p_{12}{}^2) + \\
& +\frac{C_{12}C_{23}}{I_2}\dot{\theta}_{23}(T_{02})\Big]\sin p_1 t_2 - \frac{1}{p_2}\left[C_{12}\dot{\theta}_{12}(T_{02})(p_1{}^2 - p_{12}{}^2) + \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{23}(T_{02})\right]\Bigg\}\sin p_2 t_2 + \frac{I_2 + I_3}{I_0}m_{12},
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
M_{23}(t_2) = C_{23}\theta_{23}(t_2) = \frac{1}{p_1{}^2 - p_{12}{}^2}\Bigg\{ & \left[C_{12}\theta_{12}(T_{02}) + p_{23}{}^2 \frac{I_1 + I_2}{I_0}m_{23}\right]\cos p_1 t_2 - \\
& -\left[\frac{C_{12}C_{23}}{I_2}\theta_{12}(T_{02}) + p_{23}{}^2 M_b - p_1{}^2 \frac{I_1 + I_2}{I_0}m_{23}\right]\cos p_2 t_2 + \frac{1}{p_1}C_{23}\dot{\theta}_{23}(T_{02})(p_2{}^2 - p_{23}{}^2) + \\
& +\frac{C_{12}C_{23}}{I_2}\dot{\theta}_{12}(T_{02})\sin p_1 t_2 - \frac{1}{p_2}\left[(p_1{}^2 - p_{23}{}^2)\dot{\theta}_{23}(T_{02}) + \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{12}(T_{02})\right]\Bigg\}\sin p_2 t_2 + \frac{I_1 + I_2}{I_0}m_{23},
\end{aligned}
\tag{26}
$$

where

$$
\left.
\begin{aligned}
& p_{12}{}^2 = \frac{1}{2}(p_{12}{}^2 + p_{23}{}^2) \pm \sqrt{\frac{1}{4}(p_{12}{}^2 + p_{23}{}^2)^2 - \frac{C_{12}C_{23}I_0}{I_1 I_2 I_3}}; \\
& p_{12}{}^2 = \frac{C_{12}(I_1 + I_2)}{I_1 I_2}; \qquad p_{23}{}^2 = \frac{C_{23}(I_2 + I_3)}{I_2 I_3}; \\
& m_{12} = \frac{I_1 + I_2}{I_2}M_a + \frac{I_1}{I_2}M_b; \qquad m_{23} = \frac{I_3}{I_2}M_a + \frac{I_2 + I_3}{I_2}M_b
\end{aligned}
\right\}.
\tag{27}
$$

Constant components of expressions (25) and (26) are elastic moments of units caused by influence of static forces. Their values can be determined on the basis of solution of the system of algebraic equations, which in its turn is obtained from expressions (16) by supposing $\theta_{12} = 0$ and $\theta_{23} = 0$.

Then

$$
\left.
\begin{aligned}
M_{12} p_{12}^2 - \frac{C_{12}}{I_2} M_{23} &= M_a p_{12}^2; \\
-M_{12} \frac{C_{23}}{I_2} + M_{23} p_{23}^2 &= M_b p_{23}^2.
\end{aligned}
\right\}, \tag{28}
$$

where $M_{12} = C_{12}\theta_{12}$, $M_{23} = C_{23}\theta_{23}$.

Maximum dynamic coefficient in elastic units after closing of the second clearance Δ_2 is determined as follows

$$
\left(K_d^{(1)}\right)_{max} = \frac{\max M_{12}(t_2)}{m_{12}}\left(\frac{I_0}{I_2 + I_3}\right); \qquad \left(K_d^{(2)}\right)_{max} = \frac{\max M_{23}(t_2)}{m_{23}}\left(\frac{I_0}{I_1 + I_2}\right). \tag{29}
$$

Maximum values of $M_{12}(t)$ and $M_{23}(t)$ can be determined by plotting graphs of variation of these moments with time on the basis of application of formulae (25) and (26).

It is necessary to know the character of variation of angular velocities of masses in the course of study of the mechanical system, then we have to apply expressions of transformation of coordinates (14), (25) and (26). These expressions should be divided at first by C_{12} and C_{23}, and then differentiated with respect to time. Then we will have

$$
\left.
\begin{aligned}
\omega_1(t_2) = \frac{d\varphi}{dt_2} &= \varepsilon_{ar}^{(3)} t_2 + \dot{U}(T_{02}) - \frac{p_1}{I_0(p_2^2 - p_1^2)}\left\{\theta_{12}(T_{02})\left[(I_2 + I_3)(p_2^2 - p_{12}^2) + \frac{I_3}{I_2}C_{12}\right]+\right. \\
&+ \frac{m_{12}(I_2 + I_3)}{I_0}\left(\frac{I_0}{I_1} - \frac{I_2 + I_3}{C_{12}}p_1^2\right) - \frac{I_3(I_1 + I_2)}{I_0 C_{23}}p_2^2 m_{23}\right\}\sin p_1 t + \\
&+ \frac{p_2}{I_0(p_2^2 - p_1^2)}\left\{\theta_{12}(T_{02})\left[(I_2 + I_3)(p_1^2 - p_{12}^2) + \frac{I_3}{I_2}C_{12}\right]+\frac{m_{12}(I_2 + I_3)}{I_0}\left(\frac{I_0}{I_1} - \frac{I_2 + I_3}{C_{12}}p_1^2\right) - \right. \\
&- \frac{I_3(I_1 + I_2)}{I_0 C_{23}}p_2^2 m_{12}\right\}\sin p_2 t + \frac{1}{I_0(p_2^2 - p_1^2)}\left\{\dot{\theta}_{12}(T_{02})\left[(I_2 + I_3)(p_2^2 - p_{12}^2) + \frac{I_3}{I_2}C_{12}\right]+\right. \\
&+ \dot{\theta}_{23}(T_{02})\left[\frac{C_{23}(I_2 + I_3)}{I_2} + I_3(p_2 - p_{23}^2)\right]\right\}\cos p_1 t - \\
&- \frac{1}{I_0(p_2^2 - p_1^2)}\left\{\dot{\theta}_{12}(T_{02})\left[(I_2 + I_3)(p_2^2 - p_{12}^2) + \frac{I_3}{I_2}C_{12}\right]+\dot{\theta}_{23}(T_{02})\left[\frac{C_{23}(I_2 + I_3)}{I_2} + I_3(p_2^2 - p_{23}^2)\right]\right\}
\end{aligned}
\right\}. \tag{30}
$$

Similarly $\omega_2(t_2)$, $\omega_3(t_2)$ can be determined.

We determine the moments of elastic forces $M_{12}(t)$ and $M_{23}(t)$ for the case when there is no clearance in the second elastic unit. In this case differential equations (13), after closing of clearance Δ_1 in the first unit, must satisfy the following initial conditions:

$$
\varphi_1(t_1) = \varphi_2(t_1) = \varphi_3(t_1); \quad \dot{\varphi}_1(t_1) = \omega_{1init}; \quad \dot{\varphi}_2(t_1) = \dot{\varphi}_3(t_1) = 0. \tag{31}
$$

After solution of differential equations (13) with such initial conditions we find:

$$
\left.
\begin{aligned}
M_{12}(t_1) = \frac{1}{p_2^2 - p_1^2}&\left\{\left(M_a p_{12}^2 - p_2^2 \frac{I_1 + I_3}{I_0}m_{12}\right)\cos p_1 t_1 - \left(M_a p_{12}^2 - p_1^2 \frac{I_2 + I_3}{I_0}m_{12}\right)\cos p_2 t_1 + \right. \\
&+ \frac{C_{12}\omega_{1init}}{p_1}\left(p_2^2 - p_{12}^2\right)\sin p_1 t_1 - \frac{C_{12}\omega_{1init}}{p_2}\left(p_1^2 - p_{12}^2\right)\sin p_2 t_1\right\}+\frac{I_2 + I_3}{I_0}m_{12}.
\end{aligned}
\right\}, \tag{32}
$$

$$M_{23}(t_1) = \frac{1}{p_2^2 - p_1^2} \left\{ \left(M_b\, p_{23}^2 - p_2^2 \frac{I_1 + I_2}{I_0} m_{23} \right) \cos p_1 t_1 - \left(M_b\, p_{23}^2 - p_1^2 \frac{I_1 + I_2}{I_0} m_{23} \right) \cos p_2 t_1 + \right.$$
$$\left. + \frac{C_{12}\, C_{23}\, \omega_{linit}}{\theta_2 p_1} \sin p_1 t_1 - \frac{C_{12}\, C_{23}\, \omega_{linit}}{\theta_2 p_2} \sin p_2 t_1 \right\} + \frac{I_1 + I_2}{I_0} m_{23}. \tag{33}$$

Comparison of expressions (25), (26) and (32), (33) shows that these latter can be obtained from formulae (25) and (26), assuming that $\theta_{12}(T_{02}) = 0, \dot{\theta}_{12}(T_{02}) = \omega_1 init, \theta_{23}$ which corresponds to meeting the initial conditions (31) at moment t_1.

Consider calculation of dynamical loads with the example of the primary mill of the Krivoi-Rog Metallurgical Plant, which is characterized by the following parameters: $I_1 = 9.8 \text{ton} \cdot \text{m} \cdot \text{sec}^2$ – moment of inertia of the electromotor rotor; $I_2 = 0.56 \text{ton} \cdot \text{m} \cdot \text{sec}^2$ – moment of inertia of rollers of the gear cage; $I_3 = 0.50 \text{ton} \cdot \text{m} \cdot \text{sec}^2$ – moment of inertia of rollers reduced on the electromotor shaft; $C_{12} = 2 \cdot 10^4 \text{ton} \cdot \text{m}$ – stiffness of the electromotor shaft; $C_{23} = 1.1 \cdot 10^4 \text{ton} \cdot \text{m}$ – reduced rigidity of spindles; $\Delta_1 = 0.01$ rad – value of clearance in the first section.

Exploitation of rolling mills shows that clearances in spindle joints increase gradually as the result of wear of the washers surfaces. In this connection it is interesting to know how dynamical loads vary depending on the increase of clearances in main drive of the rolling mill. For this purpose we assume that the first clearance is constant (Δ_1=const) and the second clearance varies in the range $\Delta_2 = (0.02 - 005)$ rad; the starting moment of the electromotor M_1=40ton·m, and moments of technological load (rolling moment), applied to the second and third masses, respectively are M_2=2ton·m; M_3= 4ton·m.

1. Determine the angular acceleration and velocity of the first mass (rotor) after closing of the first clearance

$$\varepsilon_0 = \frac{M_1}{I_1} \cong 4.08 \; \frac{1}{\text{sec}^2}; \; \omega_{limit} = \sqrt{2\varepsilon_0 \Delta_1} = 0.286 \; \frac{1}{\text{sec}^2}; T_{01} = \sqrt{2\Delta_1 / \varepsilon_0} \cong 0.07 \text{sec}$$

2. Frequency of proper oscillations p of the two-mass system elastic moment M_a and average acceleration after closing of the first clearance

$$p = \sqrt{\frac{C_{12}(I_1 + I_2)}{I_1 I_2}} \cong 194.3 \; \frac{1}{\text{sec}}; \; M_a = \frac{M_1 I_2 + M_2 I_1}{I_1 + I_2} \cong 4.05 \text{ton} \cdot \text{m}; \; \varepsilon_{av}^{(2)} = \frac{M_1 - M_2}{I_1 + I_2} \cong 3.67 \; \frac{1}{\text{sec}^2}$$

3. Determine the time T_{02} necessary for closing of the second clearance with the use of equation (9)

$$-\frac{4.05}{0.56 \cdot 194.3^2}(1 - \cos 194.3\, T_{02}) - \frac{9.8 \cdot 0.286}{10.36 \cdot 194.3} \sin 194.3\, T_{02} + \frac{9.8 \cdot 0.286}{10.36} T_{02} + 1.835 T_{02}^2 = \begin{cases} 0.02 \\ 0.03 \\ 0.05 \end{cases} \tag{34}$$

At first we determine approximate values of T_{02} with the use of formula (11):

$$T_{02}\big|_{\Delta=0.02} = 0.054; \quad T_{02}\big|_{\Delta=0.03} = 0.073; \quad T_{02}\big|_{\Delta=0.05} = 0.105. \tag{35}$$

We find more exact values with the use of more precise transcendental equation (9):

$$T_{02}\big|_{\Delta=0.02} = 0.0526; \quad T_{02}\big|_{\Delta=0.03} = 0.0724; \quad T_{02}\big|_{\Delta=0.05} = 0.1065.$$

Determine the values of $\theta_{12}(T_{02}), \theta_{23}(T_{02}), \dot{\theta}(T_{02}), \dot{\theta}_{23}(T_{02})$ with the use of expressions (16,17,19,20) and then using formulae (25), (26) we obtain the following expressions for calculation of moments of elastic forces $M_{12}(t)$ and $M_{23}(t)$:

$$M_{12}^{0.02}(t_2) = 10.96 - 18.52\cos\ 114.6t_2 + 23.17\sin\ 114.6t_2 - 6.79\cos\ 257.4t_2 - 27.96\sin\ 257.4t_2;$$

$$M_{12}^{0.03}(t_2) = 10.96 + 6.93\cos\ 114.6t_2 + 35.13\sin\ 114.6t_2 + 15\cos\ 257.4t_2 - 9.65\sin\ 257.4t_2; \tag{36}$$

$$M_{12}^{0.05}(t_2) = 10.96 + 7.5\cos\ 114.6t_2 + 54.24\sin\ 114.6t_2 + 15.46\cos\ 257.4t_2 - 16.28\sin\ 257.4t_2.$$

$$M_{213}^{0.02}(t_2) = 3.85 - 8.42\cos114.6t_2 + 15.64\sin114.6t_2 + 4.57\cos257.4t_2 + 22.21\sin257.6t_2;$$

$$M_{23}^{0.03}(t_2) = 3.85 + 9.1\cos114.6t_2 + 26.43\sin114.6t_2 - 12.95\cos257.4t_2 + 8.83\sin257.4t_2; \tag{37}$$

$$M_{23}^{0.05}(t_2) = 3.85 + 9.47\cos114.6t_2 + 37.05\sin114.6t_2 - 13.32\cos257.4t_2 + 10.67\sin t_2.$$

Determine the values of $M_{12}(t)$ and $M_{23}(t)$ for the case when there is a clearance in elastic unit C_{12}, and a clearance is lacking in unit C_{23}, i.e. $\Delta_2 = 0$. Then, using expressions (32) and (33) we have

$$M_{12}(t_1) = 10.96 - 10.78\cos114.6t_1 + 26.78\sin114.6t_1 - 0.17\cos257.4t_1 + 10\sin257.4t_1, \tag{38}$$

$$M_{23}(t_1) = 3.85 - 3.1\cos114.6t_1 + 18.46\sin114.6t_1 - 0.75\cos257.4t_1 - 8.22\sin257.4t_1. \tag{39}$$

We accomplish numerical calculation of expressions (37) and (38) in the time intervals $0.0526\,\text{sec} \le t_1 \le 0.25\,\text{sec}$; $0.0724\,\text{sec} \le t_2 \le 0.25\,\text{sec}$; $0.1065\,\text{sec} \le t_2 \le 0.25\,\text{sec}$ and calculation of expressions (38) and (39) in the range of time $0 \le t_1 \le 0.25\,\text{sec}$. As a result of these calculations we determine the maximum magnitudes of $M_{12}^{\Delta_2}, M_{23}^{\Delta_2}, M_{12}, M_{23}$.

Then magnitudes of $\left(\dfrac{M_{12}^{\Delta_2}}{M_{12}}\right)_{\max}$ and $\left(\dfrac{M_{23}^{\Delta_2}}{M_{23}}\right)_{\max}$, determining the rate of increase of elastic moments in comparison with the values of static moments caused by the clearance in the second unit C_{23} are calculated. The results of calculations are given in the Table:

Δ_2	$(M_{12}^{\Delta_2}/M_{12})_{\max}$	$(M_{23}^{\Delta_2}/M_{23})_{\max}$
0.02	1.39	1.43
0.03	1.32	1.55
0.05	1.76	1.92

Analysis of the Table shows that the existence of the clearance in the second elastic unit (C_{23}) (spindle joint) causes significant increase of maximums of moments of elastic forces $M_{12}(t)$ and $M_{23}(t)$. Thereat, the rate of increase of the dynamic coefficient in the second elastic unit is greater than in the first (C_{12}) unit.

The dynamic coefficient in elastic unit C_{23} increases with the increase of the clearance Δ_2 and this coefficient decreases partially in unit C_{12}. This points to the fact that the maximum values of moments of elastic forces depend not only on the values of clearances but also on the initial conditions at the moment of closing of the clearance; namely, on the velocity of the mass moving in the zone of clearance. It is natural that the higher is the velocity at percussion of masses at the moment of closing of the clearance, the greater will be the magnitudes of percussive forces and maximum values of moments of elastic forces generated by them.

Maximum values of dynamic coefficients given in the Table are in full correspondence with the data obtained as the result of experimental research into dynamic loads of transmissions of various primary mills [1-2].

It is expedient to use the results and conclusions obtained in the present paper both in designing new rolling facilities and in the realization of optimal regulation of rolling processes by the operator in transitional (at seizure of metal by rollers and at braking of rollers) regimes.

მექანიკა

ღრეჩოების ზეგავლენის გამოკვლევა სამანქანო სისტემების დინამიკურ დატვირთვებზე

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შემოთავაზებულია სამანქანო აგრეგატების ტრანსმისიებში დინამიკური მახასიათებლების განგარიშების განზოგადოებული მეთოდიკა მანქანის გაშვების სხვადასხვა ეტაპზე მთავარი ელექტროამძრავის ძალოვან გადაცემებში ღრეჩოების თანაარსებობის გათვალისწინებით. დადგენილია ანალიზური დამოკიდებულება ღრეჩოების სიდიდეებსა და მათი გავლენით გამოწვეულ დამატებით (დარტყმითი ხასიათის) ძალვებს შორის.

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