*Informatics*

# Bicriteria Fuzzy Vehicle Routing Problem for Extreme Environment

## Gia Sirbiladze*, Bezhan Ghvaberidze*, Bidzina Matsaberidze*

*Faculty of Exact and Natural Sciences, I.Javakhishvili Tbilisi State University, Tbilisi*

(Presented by Academy Member Mindia Salukvadze)

**ABSTRACT. Route planning problems are among the activities that have the highest impact on logistical planning, transport and distribution because of their effects on efficiency in resource management, service levels, and client satisfaction. The difficulty of movement between different customers and other problems cause the uncertainty of time of movement. In this paper this uncertainty is presented by a possibility measure and a fuzzy set. A new multiple criteria fuzzy optimization approach for the solution of the optimal vehicle routing problem is considered. The min-max bicriteria Vehicle Routing Problem is considered in fuzzy environment. Based on the algorithm of preferences on the first phase the sample of the so-called "promising" routes is selected. A new subjective criterion – maximization of expectation of reliability of movement on closed routes is constructed. Our problem is reduced to the min-max bicriteria fuzzy partitioning problem. Consequently, Fuzzy Expectation and Dependent Chance Programming Problems are constructed. Two phase scheme is used for the numerical solution of the Fuzzy Vehicle Routing Problem. Two possible solutions of this scheme are considered.** © 2014 Bull. Georg. Natl. Acad. Sci.

*Key words: vehicle routing problem, multiple-criteria optimization, possibility theory, fuzzy partitioning problem*

The timely distribution of goods to different customers is very complicated in extreme and difficult situations, such as: roads with overloaded traffic, public demonstrations and strikes, slippery, snowy roads or roads with less visibility, damaged roads because of natural disasters, earthquakes and other causes, etc. In such case it is important to assess the reliability and possibility levels of movement on routes. Of course, this changes the movement time as well. It becomes uncertain. Using the software for route planning in stationary environment has less sense in such cases for distribution companies. They should use the intelligent support system for optimal route planning in complex and extreme situations, which would enable experts to introduce corrections to the route planning problem based on experts' evaluations. First of all, they will consider the criterion of reliability of route together with other objective information. In such technologies experts become involved in data gathering process, and create the possibility levels of movement (travel) between customers based on embedded knowledge engineering methods and algorithms.

Let us denote these possibility levels by $\| \pi_{ij} \|$, where $\pi_{ij}$ is a possibility (conditional) level [1] of moving from i-th customer to j-th customer. Thus, expert knowledge engineering serves the optimal route planning in extreme situations. We plan to develop a control system based on fuzzy statistics to evaluate this possibility levels using results of our research [2,3].

Possibility theory was proposed by L.A. Zadeh in 1978 [4] and developed by D. Dubois and H. Prade in 1988. Since the 1980s, the possibility theory has become more and more important in the decision and optimization field and several methods have been developed to solve possibilistic programming problems ([5-12] and others). Our aim is to create possibilistic environment of knowledge engineering for optimal vehicle routing problem when movement on roads is difficult. Based on this and other objective information the possibilistic criterion is developed – called the reliability of moving on closed routes.

The systems approach and analysis play determining role in the vehicle routing problem(VRP) ([13-15] and others). The classical VRP is developed by many well-known authors ([15] and others). Here we present a new vision of the fuzzy vehicle routing problem (FVRP), which is different from the approaches given in other researches. This new problem is connected with difficulties of optimal routing of vehicles in different extreme situations.

## Construction of bicriteria optimization problem

We consider the following problem of optimal routing of vehicles as a main problem in extreme situations. Let the set of geographical points (called customers later on) $I = \{1, 2, \ldots, n+1\}$ be given, where $n+1$-th node is depot. Other customers are supplied from depot by vehicles with uniform goods. The demand of goods from customers is known, as well as maximum load and mileage of the vehicles. The problem is the following (first criterion): *It is necessary to deliver the demanded goods to customers by vehicles in such way that total distance traveled by vehicles is minimal.* It is meant that the demand of goods by each customer is much less than maximum load of vehicles.

Suppose, that $C = \| c_{ij} \|$, $i, j \in I$ is a matrix of positive real numbers and represents the distances between customers; Q and D are real numbers – the constraints of load and mileage of vehicles. $P_i$, $i = 1, 2, \ldots, n$ are also real numbers and represent the demand for goods by $i$-th customer $1 \le P_i < Q$, $i = 1, 2, \ldots, n$.

We have to find such closed set of routes $\{ M_k \}$, $k = 1, 2, \ldots, m_j$, t, $M_k = \{ n+1, i_1^k, \ldots, i_{\ell_k}^k, n+1 \}$, $i_j^k \in \{ 1, 2, \ldots, n \}$, $j = 1, \ldots, \ell_k$, $1 \le \ell_k \le n$ (m and $\ell_k$ are not fixed in advance) that satisfy the constraints

$$\bigcup_{k=1}^{m} M_k = I ; \quad M_k \cap M_q = \{ n+1 \} ,$$

$$k, q \in \{ 1, 2, \ldots, m \} , \quad k \ne q ; \sum_{j=1}^{\ell_k} P_{i_j^k} \le Q ; \tag{1}$$

$$c_k = c_{n+1, i_1^k} + c_{i_{\ell_k}^k, n+1} + \sum_{j=1}^{\ell_k - 1} c_{i_j^k, i_{j+1}^k} \le D , \quad k = 1, 2, \ldots, m ;$$

and that have minimal total distance (objective function, first criterion)

$$\sum_{k=1}^{m} c_k = \sum_{k=1}^{m} \left( c_{n+1,\, i_1^k} + c_{i_{\ell_k}^k,\, n+1} + \sum_{j=1}^{\ell_k-1} c_{i_j^k,\, i_{j+1}^k} \right). \tag{2}$$

Presented problem corresponds to some mathematical model of above presented problems, connected with distribution of uniform goods in small-sized portions between different customers. It is a complex combinatorial optimization problem and is known as NP-hard [16,20].

The difficulty of movement between different customers and other problems cause the uncertainty of time of movement. Suppose, that the expert evaluation of time for moving from i-th customer to j-th customer is represented by nonnegative normalized fuzzy-triangular numbers $\tilde{t}_{ij} = \left( t_{ij}^{(1)}, t_{ij}^{(2)}, t_{ij}^{(3)} \right)$ ([1], etc.), the membership function of which is defined by formula (3).

$$\mu_{\tilde{t}_{ij}}(t) = \begin{cases} 0, & t \le t_{ij}^{(1)}; \\[2mm] \dfrac{t - t_{ij}^{(1)}}{t_{ij}^{(2)} - t_{ij}^{(1)}}, & t_{ij}^{(1)} < t \le t_{ij}^{(2)}; \\[3mm] \dfrac{t_{ij}^{(3)} - t}{t_{ij}^{(3)} - t_{ij}^{(2)}}, & t_{ij}^{(2)} < t \le t_{ij}^{(3)}; \\[3mm] 0, & t > t_{ij}^{(3)}. \end{cases} \tag{3}$$

Then the movement time on closed route $M_k$ will also be fuzzy-triangular number:

$$\tilde{t}_k = \sum_{j=1}^{\ell_k-1} \tilde{t}_{i_j^k,\, i_{j+1}^k} + \tilde{t}_{n+1,\, i_1^k} + \tilde{t}_{i_{\ell_k}^k,\, n+1} \tag{4}$$

In usual conditions $C_k$ and $\tilde{t}_k$ are identical, but in our case $\tilde{t}_k$ shouldn't depend much on $C_k$. In extreme situations the reliability of moving on route $M_k$ is determined by smallness of $\tilde{t}_k$. So we introduce the following parameter normalized in [0,1] as a measure of reliability of route $M_k$:

$$\tilde{\delta}_k = \frac{1}{1 + \left( \tilde{t}_k \right)^{\gamma_1}} . \tag{5}$$

Also we introduce the objective weight of moving from i-th to j-th customer on route $M_k$:

$$w_{ij}^{(k)} = \frac{(1/C_{ij})^{\gamma_2}}{\sum\limits_{\mu,\nu} (1/C_{\mu\nu})^{\gamma_2}} , \quad i, j, \mu, \nu \in M_k . \tag{6}$$

$\gamma_1$ and $\gamma_2$ are positive numbers, selected using the principle of closeness to experimental data. We have to consider $w_{ij}^{(k)}$ weights and $\pi_{ij}$ possibilities, when constructing the weighted possibility level of movement on route $M_k$:

$$\pi_k^0 = \sum_{j=1}^{\ell_k-1} w_{i_j^k,\, i_{j+1}^k}^{(k)} \cdot \pi_{i_j^k,\, i_{j+1}^k} + w_{n+1,\, i_1^k}^{(k)} \cdot \pi_{n+1,\, i_1^k} + w_{i_{\ell_k}^k,\, n+1}^{(k)} \cdot \pi_{i_{\ell_k}^k,\, n+1} , \tag{7}$$

Suppose $\bar{M} = \{ M_k \}_{k=1}^{m}$ is a set of some closed routes, satisfying the (1)-(2) constraints of the main problem. To construct the reliability measure of routes $\{ M_k \}_{k=1}^{m}$ we use Choquet integral ([1,3] and others), which

condenses the reliabilities $\tilde{\delta}_k$ of routes $M_k$, $k=1,2,\ldots,m$ and their possibility distribution $\pi_k = \dfrac{\pi_k^0}{\max\limits_{k=1,m}\pi_k^0}$ on

the set $\bar{M} = \{M_k\}_{k=1}^m$. As known, this later creates possibility uncertainty with possibility measure: $Pos: 2^{\bar{M}} \to [0,1]$ [2, 7]

$$Pos\left(\{M_{j_1}, M_{j_2}, \ldots, M_{j_p}\}\right) = \max_{v=1,p} \pi(M_{j_v}) = \max_{v=1,p} \pi_{j_v}, \tag{8}$$

if $\{M_{j_1}, M_{j_2}, \ldots, M_{j_p}\} \subseteq \bar{M}$.

In fuzzy statistics the Choquet integral defined on finite set is known as Monotone Expectation (ME) [10]. Here we introduce the notion of Monotone Expectation of reliability of the set $\bar{M}$ of closed routes with respect to possibility measure:

$$ME\left(\tilde{\delta}_1, \tilde{\delta}_2, \ldots, \tilde{\delta}_m\right) \overset{\Delta}{=} \int_0^1 Pos\left(\{M_j \in \bar{M} / \tilde{\delta}_j \geq \alpha\}\right) d\alpha =$$

$$= \sum_{i=1}^m \left[ Pos\left(\{M_{j(1)}, M_{j(2)}, \ldots, M_{j(i)}\}\right) - Pos\left(\{M_{j(1)}, M_{j(2)}, \ldots, M_{j(i-1)}\}\right) \right] \cdot \tilde{\delta}_{j(i)} = \tag{9}$$

$$= \sum_{i=1}^m \left[ \max_{j=1,i} \pi_{j(i)} - \max_{j=1,i-1} \pi_{j(i-1)} \right] \cdot \tilde{\delta}_{j(i)},$$

where $\pi_{j(0)} = 0$; $Pos\left(\{M_{j(0)}\}\right) \equiv 0$; $\tilde{\delta}_{j(1)} \geq \tilde{\delta}_{j(2)} \geq \cdots \geq \tilde{\delta}_{j(m)}$, – is the permutation of $\{\tilde{\delta}_1, \tilde{\delta}_2, \ldots, \tilde{\delta}_m\}$ which decreasingly orders the reliability levels. Let's denote: $P^{(i)} \equiv \max\limits_{v=1,i} \pi_{j(v)} - \max\limits_{v=1,i-1} \pi_{j(i-1)}$. It can be easily checked that $\{P^{(i)}\}_{i=1}^m$ values have properties of probability distribution on $\bar{M}$. In this case (9) can be written as:

$$ME\{\tilde{\delta}_1, \tilde{\delta}_2, \ldots, \tilde{\delta}_m\} = \sum_{i=1}^m P^{(i)} \cdot \tilde{\delta}_{j(i)}, \tag{10}$$

This is a second criterion– *maximization of monotone expectation of reliability of movement on the closed routes* $\{M_k\}_{k=1}^m$. As in previous case (formula (7)) we plan to engineer new aggregation instruments in possibility theory.

The objective of our research is planned to be implemented to two-phase scheme of solving bicriteria problem.

Phase I: Allowed closed routes (which satisfy (1)-(2) constraints) should also satisfy additional criteria: 1) The time of movement on closed route $M_k$ must not exceed some maximum limit $\tilde{t}_k \leq \tilde{t}_{\max}$, and 2) The possibility level of movement on route $M_k$ must exceed some minimum level: $\pi_k \geq \pi_{\min}$ ( $\pi_{\min}$ and $\tilde{t}_{\max}$ are predefined by expert group based on their needs).

Among all allowable closed routes (in case of large dimension of problem, when the number of allowable routes is in order of $10^4 - 10^6$) during the first phase the sample of so-called "promising" routes are selected

$$\bar{M} = \{M_1, M_2, \ldots, M_q\},$$

which satisfy above criteria (the number of such promising routes should be in order of $10^3$ –due to limitations of second phase solutions). We plan to make this selection using the specially created algorithm of

preferences based on the results from [13,14]. Note that this phase will be necessary only in cases when problem's dimension is large.

Phase II: For the promising routes $\bar{M} = \{M_1, M_2, \ldots, M_q\}$ selected during 1st phase, new bicriteria problem will be solved. The two criteria are minimization of total traveled distance and maximization of reliability of routes. The problem will be stated as a fuzzy-partitioning problem (FPP) [2,8].

Our further discussion concerns to a fuzzy partitioning problem(FPP). Some questions of FPP with possibility-probability uncertainty have already been investigated by the authors of this work ([2,8] and others). Now we consider a new variant of a presentation of the optimal FPP for the solution of the FVRP presented here.

Let $A = \| a_{ij} \|_{n \times q}$ be an incidence matrix, and $a_{ij} = 1$ if route $M_j$ goes through customer i and $a_{ij} = 0$ otherwise.

All subset of routes $M' = \{M_{j_1}, M_{j_2}, \ldots, M_{j_p}\} \subseteq \bar{M}$ is presented by means of its characteristic vector which has a component $x_j = 1$ if the route $M_j$ is contained in the subset $M'$ and $x_j = 0$ otherwise. As determined in previous section, each route $M_j$ has the level of possibility of movement on it - $\pi_j$, level of reliability $\tilde{\delta}_j$ and, of course, the total distance of route $C_k$. Then we create the distance vector $\bar{C} = \{c_1, c_2, \ldots, c_q\}$ of route $\bar{M}$; $\tilde{\delta} = \{\tilde{\delta}_1, \tilde{\delta}_2, \ldots, \tilde{\delta}_q\}$ – is the vector of reliabilities; $\bar{\pi} = \{\pi_1, \pi_2, \ldots, \pi_q\}$ – the vector of possibilities (possibility distribution on $\bar{M}$);Also $\bar{x} = \{x_1, x_2, \ldots, x_q\} \in \{0, 1\}^q$ – Boolean vector. Then criteria (2) in partitioning problem sounds this way: the set of routes $M'$ is called the partitioning of sets of customers I (otherwise we can say that for any customer from I there exist only one route from $M'$ that travels through that customer, except the depot, which is the starting and ending point of all routes), if

$$\bigcup_{v=1}^{p} M_{j_v} = I, \ M_{j_v} \bigcap M_{j_\mu} = \{n+1\}, \ j_v, j_\mu \in \{j_1, j_2, \ldots, j_p\}. \tag{11}$$

The partitioning condition is similar to the following system of linear equations:

$$A\bar{x} = \bar{e}, \tag{12}$$

where $\bar{e} = (1, 1, \ldots, 1)$ is a vector consisting of 1s. Note that because of 1st phase, the partitioning routes satisfy the following conditions: if $M_k \in M'$, then

$$\sum_{j=1}^{\ell_k} P_{i_j k} \leq Q, \ c_k \leq D, \ \pi_k \geq \pi_{\min}, \ \tilde{\delta}_k \geq 1 / \left(1 + \left(\tilde{t}_{\max}\right)^{\gamma_2}\right). \tag{13}$$

Because of (1)-(2) for each partitioning $M'$, we construct the objective function for the total distance of route:

$$\sum_{i=1}^{n} c_i x_i, \ A\bar{x} = \bar{e} \tag{14}$$

and based on (10) – the objective function of their reliability:

$$\sum_{i=1}^{n} P^{(i)} \cdot \tilde{\delta}_{j(i)} x_{j(i)}; \tag{15}$$

So, we consider the Min-max bicriteria fuzzy-partitioning problem:

$$f_1 \equiv \sum_{i=1}^{q} c_i x_i \Rightarrow \min \qquad (16)$$

*minimization of the distances of routes in partitioning* and

$$f_2 \equiv \sum_{i=1}^{q} P^{(i)} \cdot \tilde{\delta}_{j(i)} x_{j(i)} \Rightarrow \max \qquad (17)$$

*maximization of monotone expectation of reliability of routes* in partitioning with linear constraints

$$A\bar{x} = \bar{e}, \; x_i \in \{0,1\}. \qquad (18)$$

Note that the monotone expectation of reliability of routes in partitioning is triangular fuzzy number (TFN) and maximization of (17) criterion must be understood as fuzzy-maximization. In addition of (17) we will consider other criterion from Possibilistic Programming Approach. We present one of them here:

If $\mu(x)$, $x \in R$ is the membership function of a fuzzy value $\xi$ then $Pos(\xi \le x) = \sup_{u \le x} \mu(u)$ [5,17]. For a TFN $\xi = (a_1, a_2, a_3)$ we have the following equations:

$$Pos(\xi \le x) = \begin{cases} 1, & x > a_2; \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2; \\ 0, & x < a_1. \end{cases} \qquad (19)$$

Let us consider the following criteria the reliability of routes in partitioning: $Pos(f_2 \ge \bar{f}_2)$. Obviously this equation is written down using linear sum of parameters of triangular numbers $\tilde{\delta}_1, \tilde{\delta}_2, \ldots, \tilde{\delta}_q$, meaning that we fix the possibility of the reliability of routes in partitioning to be higher than some apriority defined the minimal level $\bar{f}_2$. Analogously to [18], we construct our dependent chance programming problem (DCPP) with objective criteria (16), the purpose of which is to select the partition with the maximum possibility of the monotone expectation of the reliability of partition

$$\begin{cases} \min \sum_{i} c_i x_i; \\ \max Pos(f_2 \ge \bar{f}_2); \\ A\bar{x} = \bar{e}; \\ x_i \in \{0,1\}. \end{cases} \qquad (20)$$

For solving the bicriteria problems (16)-(18) or (20), two approaches will be considered in our future investigations:

1) The method of compromise will be created for bicriteria problems (16)–(18) or (20), based on the idea of ordering of criteria.

2) Scaling of (16)–(18) or (20) bicriteria problems and reducing the problem to classical partitioning problem $f = \lambda f_1 + (1 - \lambda) f_2$, $(0 < \lambda < 1)$ with one criterion. The exact solution of classic partitioning problem will be realized by D. Knuth Algorithm of Dancing Links-X (DLX) [18].

## Conclusion

A new multiple criteria optimization approach for the solution of the optimal vehicle routing problem is considered. This problem is reduced on the Min-max bicriteria fuzzy partitioning problem. Consequently, Fuzzy Expectation and Dependent Chance Programming Problems are constructed. Two phase scheme is used for the numerical solution of the FVRP.

Note that to obtain possibilistic levels of movement on routes, in future we plan to construct new generalized possibilistic operators AsPOWA, AsFPOWA and others, which are generalizations of OWA operator [19] for aggregating imprecision and uncertainty of information. Their information measures will be studied as well, such as: Orness, Balans, Entropy and others.

## Acknowledgement

*ინფორმატიკა*

# ბიკრიტერიალური ფაზი-სატრანსპორტო მარშრუტიზაციის ამოცანა ექსტრემალური გარემოსთვის

## გ. სირბილაძე, ბ. ღვაბერიძე, ბ. მაცაბერიძე

*ი. ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი*

(წარმოდგენილია აკადემიკოს მ. სალუქვაძის მიერ)

სატრანსპორტო მარშრუტიზაციის ამოცანების გადაწყვეტა განსაკუთრებით დიდ მნიშვნელობას იძენს ბოლო დროს გზებზე შექმნილი გართულებული სიტუაციების გამო: 1. ტრანსპორტით გადატვირთული გზები; 2. ექსტრემალური მოვლენები, როგორიცაა: კატასტროფები, მიწისძვრები, სხვა და სხვა მიზეზებით გამოწვეული გზებზე ცუდი ხილვადობა და სხვ. ყოველივე ეს ართულებს სატრანსპორტო მარშრუტიზაციის ამოცანების გადაწყვეტას, რადგან გზებზე გადაადგილების დრო ცალსახად აღარ განისაზღვრება გადაადგილების მანძილით. ასეთ შემთხვევაში გზებზე სატრანსპორტო საშუალებებით გადაადგილების მინიმიზაციის ამოცანასთან ერთად ჩნდება ისეთი კრიტერიუმი, როგორიცაა გადაადგილების მარშრუტების საიმედოობის მაქსიმიზაცია. ამ უკანასკნელის აგება დაკავშირებულია საექსპერტო ცოდნის ფორმირებისა და ინჟინერიის მეთოდებთან და ტექნოლოგიებთან. მეორე კრიტერიუმში წარმოშობს სატრანსპორტო მარშრუტიზაციის ბიკრიტერიალურ ამოცანას. როგორც ცნობილია, კარგადაა შესწავლილი კლასიკური სატრანსპორტო მარშრუტიზაციის ამოცანები, რომლებიც, სტაციონარული პირობების

შეზღუდვებში ძირითადად გზებზე სატრანსპორტო საშუალებებით გადაადგილების მინიმიზაციის ამოცანის გადაწყვეტას გულისხმობს. ამოცანა დისკრეტული ოპტიმიზაციის ამოცანათა კლასს მიეკუთვნება და NP-სირთულისაა. დიდი განზომილებების შემთხვევაში მის გადასაწყვეტად შექმნილია ევრისტიკული თუ სხვა ტიპის მიახლოებითი მიდგომები. ნაშრომში წარმოდგენილი ამოცანის რეალიზაციისთვის ჩვენს მიერ შეიქმნა ფაზი-სატრანსპორტო მარშრუტიზაციის ამოცანის გადაწყვეტის ახალი მიდგომა. საექსპერტო მონაცემების ბაზაზე შექმნილი მეთოდოლოგია დაფუძნებულია შესაძლებლობის თეორიის ბოლო შედეგებზე. სტატიაში განხილულია მინი-მაქს ტიპის ბიკრიტერიალური სატრანსპორტო მარშრუტიზაციის ამოცანა ექსტრემალური გარემოსთვის. აგებულია ახალი სუბიექტური კრიტერიუმი – ჩაკეტილ გზებზე გადაადგილების მოსალოდნელი სანდოობის მაქსიმიზაცია. კრიტერიუმის ასაგებად გამოყენებულია შოკეს ინტეგრალის სასრული წარმოდგენა. ამ ამოცანის რეალიზაციის ორფაზიანი სქემაა წარმოდგენილი. განხილულია ამ სქემის ორი შესაძლო გადაწყვეტა.

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